THE EFFECT OF WIND UPON THE MIXED-LAYER DEPTH

JACK E. GEARY

THE EFFECT OF WIND UPON THE MIXED-LAYER DEPTH

* * * * *

Jack E. Geary

THE EFFECT OF WIND

UPON THE MIXED-LAYER DEPTH

* * * * * *

Jack E. Geary

1961

UNITED STATES NAVAL POSTGRADUATE SCHOOL

Degree: Master of Science in Meteorology

Classification:

Thesis: UNCLASSIFIED

Abstract: Unclassified

Title of Thesis: Unclassified

Contains no proprietary information

THE EFFECT OF WIND

UPON THE MIXED-LAYER DEPTH

by

Jack E. Geary

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

IN

METEOROLOGY

United States Naval Postgraduate School
Monterey, California

1961

THE EFFECT OF WIND

UPON THE MIXED-LAYER DEPTH

by

Jack E. Geary

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

METEOROLOGY

from the

United States Naval Postgraduate School

ABSTRACT

The general problem of the mixed-layer depth of the ocean's surface layers is discussed. Wind mixing and its contribution to mixed-layer depth at ocean station Papa during the year 1958 is evaluated and discussed. A model of mixing by wind-generated wave motion is developed and is tested along with two other models; and the results are discussed.

The author wishes to express his appreciation for the advice and assistance given by Professor G. H. Jung of the U. S. Naval Post-graduate School in this investigation, and also his thanks to the Pacific Oceanic Group for their kindness in making available the necessary oceanographic data.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

A	wave amplitude
D_m	depth to which mixing takes place
E	total energy per unit area of wave surface
F	fetch length
g	acceleration of gravity
$H_{1/3}$	significant wave height
k	wave number
MLD	mixed-layer depth
T_a	air temperature
T_w	sea surface temperature
t_h	duration time of the wind
U_o	potential energy per unit area of wave surfaces
U_z	same as U_o, measured at depth z
V	wind speed
z	depth below sea surface
ρ	density of sea water
σ	angular wave frequency

1. Introduction

The phenomenon of mixing of the surface waters of much of the world's ocean area has been studied for many years. Increasing naval operational interest in the depth of the mixed layer now demands that the groundwork established by many oceanographic investigators be built upon, and that intensive effort be directed toward the goal - a usable forecast of mixed-layer depth over broad expanses of the seas.

Mixed-layer depth is the depth below the water surface to which mixing has established an essentially isothermal temperature distribution. The lower boundary of the mixed layer is the thermocline, normally a thin layer or interface of large, negative vertical temperature gradient.

Large variations of mixed-layer depth occur in space and time. Observation indicates that a full spectrum of time variation ranging from an annual cycle to short term fluctuations of a few minutes duration is to be expected at a fixed geographic location. The annual cycle is at present the most regular fluctuation of the mixed-layer depth to be identified, and is in general the largest in magnitude. The annual trend of the mixed layer is a sinusoidal wave which appears to be most closely related to the heat balance of the surface water layers, i.e. mixed-layer depth is inversely proportional to the stability of the surface water layers. This is not to say that other periodic fluctuations of mixed-layer depth such as lunar-tidal or diurnal heating and cooling are not present, but paucity of data and the magnitude of apparently random fluctuations makes their identification difficult.

Instantaneous values of mixed-layer depth may be looked upon as the result of various perturbations superimposed upon the mean annual

wave. The most significant perturbations appear to be the integrated effect of meteorological factors operating to alter the stability of the surface water layers. Winds, evaporative cooling, albedo, moisture content of the air, and insolation are a few of these meteorological factors. Added to the effects of the meterological perturbations are the apparently random motions of internal waves along the thermocline.

Geographical variations of mixed-layer depth are associated with the bathymetry of the ocean basins, the advective and mixing effects of the permanent currents and ocean tides, and the climatological regimes. The enormous problems of data collection and observational density militate against anything but qualitative estimates of the effects of space variation upon mixed-layer depth at the present time.

The purpose of this paper is to evaluate the meteorological factor of wind in its contribution to mixing of the surface waters at ocean station Papa (50N - 145W) for the year 1958.

2. The mechanism of wind-induced mixing

Wind can act directly to produce momentum transfer across the air-sea interface by means of wind-driven wave motion or by drift currents [1]. It is postulated that a given amount of wind stress will produce motion of the surface layers of water, giving rise to mixing of the water layers until stability in the region of the thermocline dampens particle motion to such an extent that equilibrium between the mixing forces and the stabilizing forces is reached, and vertical mixing can proceed no further. The amount of stress exerted by the wind upon the sea surface is known to depend upon the speed of the wind, the stability of the air, and the nature of the air-sea interface [2]. The stability of the thermocline (the limit of mixing) depends upon the wind stress [3], and the heat balance in the mixed layer.

Convective mixing is the process by which evaporation and loss of heat to the atmosphere causes the surface water layers to become unstable, resulting in overturning and mixing. Wind speed is a factor in the flux of water vapor and sensible heat across the air-sea interface. Convective mixing is, in general, most efficient when heat losses from the surface mixed layer are greatest.

In order to isolate, insofar as possible, the effect of wind-driven mechanical mixing for the purpose of study in this section, data from the months of maximum sea-surface heating were chosen from the records of the Pacific Oceanic Group for ocean station Papa, 1958, as examples of this process.

3. The relationship of wind speed to mixed-layer depth

The data were organized into time series of mixed-layer depth and wind speed, and into scatter diagrams of the same variables. Since wave motion depends on duration of wind rather than instantaneous values, the mean of the eight available daily wind observations was taken as the representative wind for each day. The selection of a representative mixed-layer depth for a day was a greater problem. In general, only two bathythermograph observations, one at 0200 and the other at 1700 Greenwich time, were available. In addition, as will be discussed later, considerable fluctuation of MLD about the daily mean value is observed, and there is no certainty that the mean of the 0200 and 1700 observations would be truly representative of the mean for the day. Therefore, the maximum observed MLD for the day was used. MLD was taken to be the depth below the sea surface at which the temperature trace of the water ceased to be isothermal; in general this is the top of the thermocline. This definition permits a zero value for MLD.

Figure 1 is a time series of mean daily wind speed and maximum observed daily MLD at ocean station Papa for the month of June, 1958. The two curves are very similar, increasing and decreasing together. Close inspection reveals that the MLD curve lags behind the wind speed curve by about 24 hours, and that most fluctuations of wind speed are followed the next day by a similar fluctuation of MLD. Another interesting characteristic of the two curves is that not only does MLD increase with increasing wind speed, it also tends toward zero as wind speed decreases.

Figure 2 is a scatter diagram obtained by plotting maximum daily

MLD against the previous day's mean wind speed for the months of June, July, August, and September, 1958. Correlation coefficients for the four months were found to be 0.94, 0.76, 0.41, and 0.68, respectively. Disregarding for the moment the August points, it is seen that the remaining points fall into two groups; those of June and July; and those of September. The August points can be divided into two groups also; those associated with the June and July group, and those associated with the September group. The separation of the points into two groups is in accord with the two different thermal regimes which are represented. The June and July points represent a period where the water is being actively heated, while the September points represent a period of beginning heat loss by the water; the August points are transitional between the two periods. The basis for this division is that the observed sea surface temperature reaches its maximum for the year in late July and begins its annual cooling trend in mid-August. This difference in regimes may explain in part why the correlation coefficients for June and July are so much larger than the August and September coefficients. In all four of the months increasing winds drive the mixed layer deeper, but only under conditions where excess heat is being supplied is there a tendency for the mixed layer to approach zero under decreasing wind conditions.

Linear regression equations were calculated for these scatter diagrams and found to be:

$MLD = 1.14\ V$ (June) (MLD in meters, V in knots)

$MLD = 0.94\ V + 1.2$ (July)

$MLD = 1.07\ V + 26.1,$ (Sept)

respectively. The small difference between the June and July slopes is attributed to the slightly more stable condition of the July water when heating was at a maximum.

The fact that mixing of the surface water can depend not only upon wind speed, but upon convection as well, is illustrated in figure 3. This figure is a scatter diagram of the same variables shown in figure 2, but for the month of November, 1958. Here a correlation coefficient of -0.58 is obtained. In November there is rapid cooling of the surface water, the temperature decreasing some four and one-half degrees fahrenheit in two weeks. In this autumn month, clearly, convection has completely overridden the effects of wind mixing alone, and is independent of wind speed.

4. Time lag of the mixed layer produced by a given wind

As pointed out in section 3, changes of MLD lag changes of wind speed. Figure 4 is a time series of wind and MLD observations taken at ocean station Papa on 16 to 19 June, 1958. MLD observations were taken hourly, while wind observations were taken at three-hour intervals. Both curves were smoothed by three-hour overlapping sums, and the MLD curve is plotted with an 18-hour lag from the wind speed observations to obtain good agreement of the fluctuations on both curves. Actually the best agreement would have been obtained by a 12-hour lag for the shallow MLD (smaller wind speed) with a larger lag for the deeper MLD values. A lag between observed wind speed and the resulting mixing of the surface water is in accord with the theory of wave generation, which requires a minimum duration time for the wind to have blown in order to produce waves of a certain size. In general, the required duration time of the wind increases with increasing size of wave produced [4] ; by analogy, the lag of MLD from wind should increase with increasing wind and mixed-layer depth.

5. Random fluctuations of MLD

The depth of the mixed layer was observed to vary several meters in a random fashion between hourly measurements. The magnitude of the fluctuations could not be made to correlate with any observed meteorological parameter, but the fluctuations did appear to increase in size with depth.

Figure 5 shows the mean hourly fluctuation of ten series of observations plotted as a function of the mean mixed-layer depth of each series. The individual series contained from 24 to 48 bathythermograph observations, and were spaced at approximately monthly intervals over the year.

Schule [5] and others have observed similar short-term fluctuations of MLD at other localities and conclude that they are caused by internal waves along the thermocline.

The importance of the random fluctuations is two-fold. A single measurement of MLD will not necessarily be representative of the mean MLD for any given day. A series of observations should be taken each day when verifying computed values of mixed-layer depth. The other important aspect of internal wave motion along the thermocline is that internal waves may provide one mechanism of vertical mixing. Ball [6] describes an experiment in which internal waves were induced along the density discontinuity between two water layers. The waves were observed to become sharp-crested and filaments of the denser water were drawn out from the sharp wave crests and mixed into the upper layer. The reverse, mixing downwards, was not observed to occur.

6. Theoretical models of wind mixing

At the start of the present investigation a search of the literature produced only one theoretical model to explain the mechanism of wind mixing. Munk and Anderson [3] developed this model incorporating the concept of vertical Austausch coefficients in the ocean, solving a system of five equations to obtain the relationship of wind speed to the depth of the mixed layer; they obtained results to within an order of magnitude of observed data, and concluded that convective mixing was at least as important as wind mixing. This is not surprising in view of Laevastu's observation [1] that the vertical Austausch coefficient varies by more than three orders of magnitude with space and time in the oceans and is at present neither measureable nor predictable.

The author approached the problem of wind mixing from the standpoint of particle motion produced by wind-driven waves. Essentially, what is required is a way to link wind speed to the motion of water particles in the surface water layers resulting from wind-produced waves. Having accomplished this, a limit remains to be set on the effectiveness of the particle motion to produce mixing. The relationship of wind speed to the various surface wave parameters (significant wave height, period, and length) was obtained using Neumann's spectrum [4], and the subsurface particle motion was in turn calculated from the theory of simple Airy waves. The maximum limit of mixing due to particle motion was assumed to be the depth were the buoyant force of the denser water below the thermocline exactly opposed the downward force of the denser particles in circular orbit. An equation (hereafter referred to as Geary's model) of the following form was derived:

$$A\sigma^2 e^{-kz} = g\left(\frac{\rho' - \rho}{\rho}\right) \qquad (1)$$

The depth at which equilibrium will be established depends upon the stability of the thermocline (the density difference across the thermocline) and upon the characteristics of the surface wave producing the motion. The necessary wave characteristics can in turn be obtained as a function of wind speed alone, assuming fully-developed seas.

During the present investigation the author became aware of the fact that Laevastu [1] had approached the problem of wind mixing from a similar viewpoint. The essential difference between the two models is that Laevastu set an arbitrary limit to the effectiveness of subsurface particle motion in accomplishing mixing. His limit was the depth where the diameter of the particle orbits was ten centimeters or less.

Neumann [7], in an extension of his surface wave spectrum to subsurface motion, derived a relationship giving the ratio of potential energy at some depth to that of the surface waves, and proposed that at the depth where the ratio was five percent or less, subsurface particle motion could be considered negligible. Neumann did not apply this concept to the problem of mixed-layer depth, but the level of wave energy present at any depth must have some influence on the work done in mixing at that depth. Consequently, Neumann's relationship was evaluated along with the models of Laevastu and Geary.

7. Fit of mixing models to observed data

The appendices contain evaluations of the three mixing models for various wind speeds. Curves 1, 2, 3, and 4 of figure 6 are plots of tables 1, 2, 3, and 4 respectively, from the appendices. The stippled area represents the field of scatter of the observed data for the months of June, July and the first half of August, 1958, some 76 observations in all.

Curve 2 is the Laevastu equation for MLD using his equation [1, pp. 70] for computing significant wave height. It can be seen that curve 2 forms an upper limit on the observed scatter of points. Curve 3 is the Laevastu equation for MLD, but using [4] for the calculation of significant wave height. It is evident that the method of calculating surface wave characteristics has an important effect upon the values obtained for mixed-layer depth.

Curve 1 is Geary's equation, and it is in surprisingly good agreement with curve 3. An explanation for this agreement cannot be suggested at the present time.

Curve 4 is the MLD equation based on Neumann's ratio of potential energies, and it can be seen that this curve fits the observed data better than the others. It may well be that a calculation of energy at depth is a better measure of mixing efficiency than are approaches using particle acceleration or the geometry of motion.

Of some interest is the fact that all of the curves deviate more from the trend of the observed scatter points at lower wind speeds, which may indicate deficiencies at low wind speeds in the wave generation equations. Munk and others have proposed a "critical wind speed" (about 13 knots), above and below which wave generation by wind proceeds differently.

8. Variation of observed data from computed values

Factors operating to cause variation of observed wind and MLD data from theoretical and empirical relationships are of three kinds. First are those that have to do with data collection and processing. Bathythermograph observations at ocean station Papa were not actually taken at a fixed location, but over an area of many square miles. The basic grid is ten nautical miles on a side, and observations are reported from several different grid positions in the course of a month. The best accuracy to which the bathythermograph data could be read is on the order of $\pm$ one meter. It is entirely possible that the differences in geographical location at which the observations were taken could account for the scatter of observed data from computed values.

A second factor, concerning only the mixing models, stems from oversimplification of the vertical mixing process and from imperfections in wind-produced wave theory. The author's assumption (appendix I) of a two-layer system is an oversimplification of actually existing density gradients. That systematic part of the variation of the mixing models from observed data may be ascribed to: 1) wind may not build waves exactly according to the theory used; 2) use of a 24-hour mean wind instead of an integrated wind could produce a systematic error of the type observed between the scatter points and curves on figure 4.

The third factor is internal wave motion. Random hourly variation of MLD was found to exist (section 5) and was calculated to be an increasing function of mixed-layer depth. Mean hourly variation of MLD increased from nearly three meters at shallow depths to about five or six meters at large MLD values. Also, the standard deviation of MLD from the

monthly mean was found to be a function of wind speed, depth, and heat balance. It is seen that internal wave motion, in addition to observational factors, could account for all of the random scatter of the plotted points.

9. Conclusions and recommendations for future research

Wind is an important factor in determining the depth to which the surface waters are mixed. The mechanism of mixing by wind-driven wave motion must exist throughout the seasons, but is dominant only when the mixed-layer depth tends toward zero in the absence of mixing. In general, this condition will occur during the summer months when heating of the surface water is maximum, and convective mixing is at a minimum.

Equations incorporating the subsurface particle motion of wind waves can be called upon to hindcast much of the observed daily and longer-interval changes in mixed-layer depth at ocean station Papa during the summer of 1958. Although turbulent mixing due to the vertical velocity shear of drift currents also must take place, this effect seems to be masked by wave mixing, since the equations for wave mixing generally give values of MLD greater than those actually observed at wind speeds greater than about 12 knots. It is possible, however, that mixing by drift currents accounts for the larger MLD values observed at shallower depths, i.e. for wind speeds less than 12 knots, or so.

The models of wave mixing should be considered as indices of mixing efficiency, not as precise physical descriptions of the vertical mixing process. No one has been able to describe the exact process, but the wave mixing approach appears to be the most promising thus far advanced.

It is difficult to say from one test which model of wave mixing as presently written, if any, will prove to be useful in a forecasting scheme. Laevastu's equations give the closest approximation to an upper limit on the observed mixing. Neumann's energy ratio fits the observed distribution better than the other equations. However Geary's model takes into

account thermocline stability, a necessary parameter if other conditions and locations are ever to be investigated.

The time lag between observed wind and resulting mixing is a provident phenomenon from a forecasting standpoint, and must be considered when verifying calculated mixed-layer depths.

Hourly and other short-term random fluctuations of MLD about a mean value prevent the forecasting of exact, instantaneous mixed-layer depths, and will have to be considered for forecasting and investigative purposes.

The most important factor requiring investigation is the problem of convective mixing. At present, this process has no model to describe its effects, yet it appears to be the singular mechanism dominating the depth to which mixing takes place during most of the annual cycle. The use of a high-speed computer for a running computation of the heat balance in the surface waters may prove to be of valuable assistance in describing short-term fluctuations of mixed-layer depth. Future research should also be applied to improving and testing models of wind mixing.

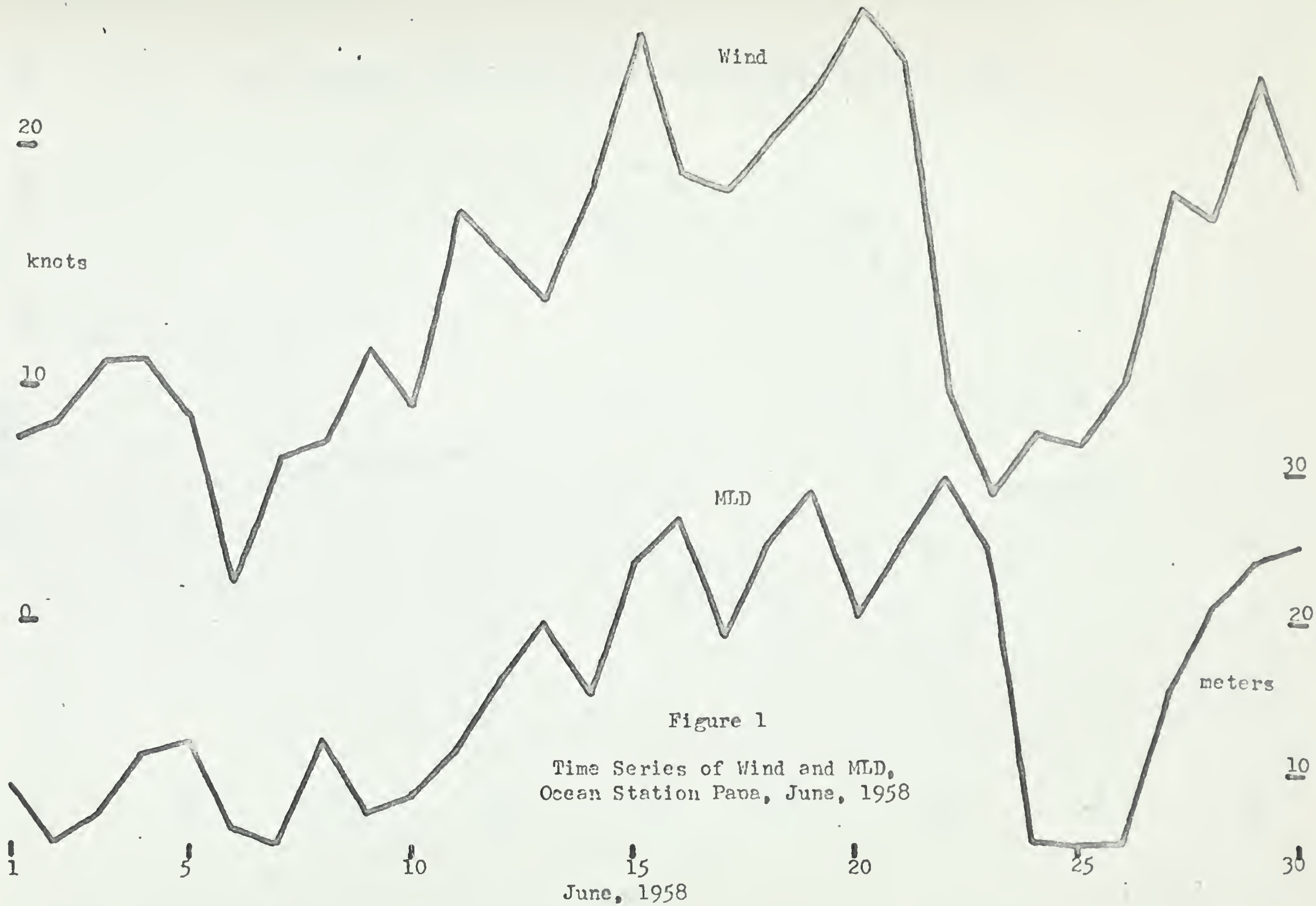

Figure 1

Time Series of Wind and MLD,
Ocean Station Papa, June, 1958

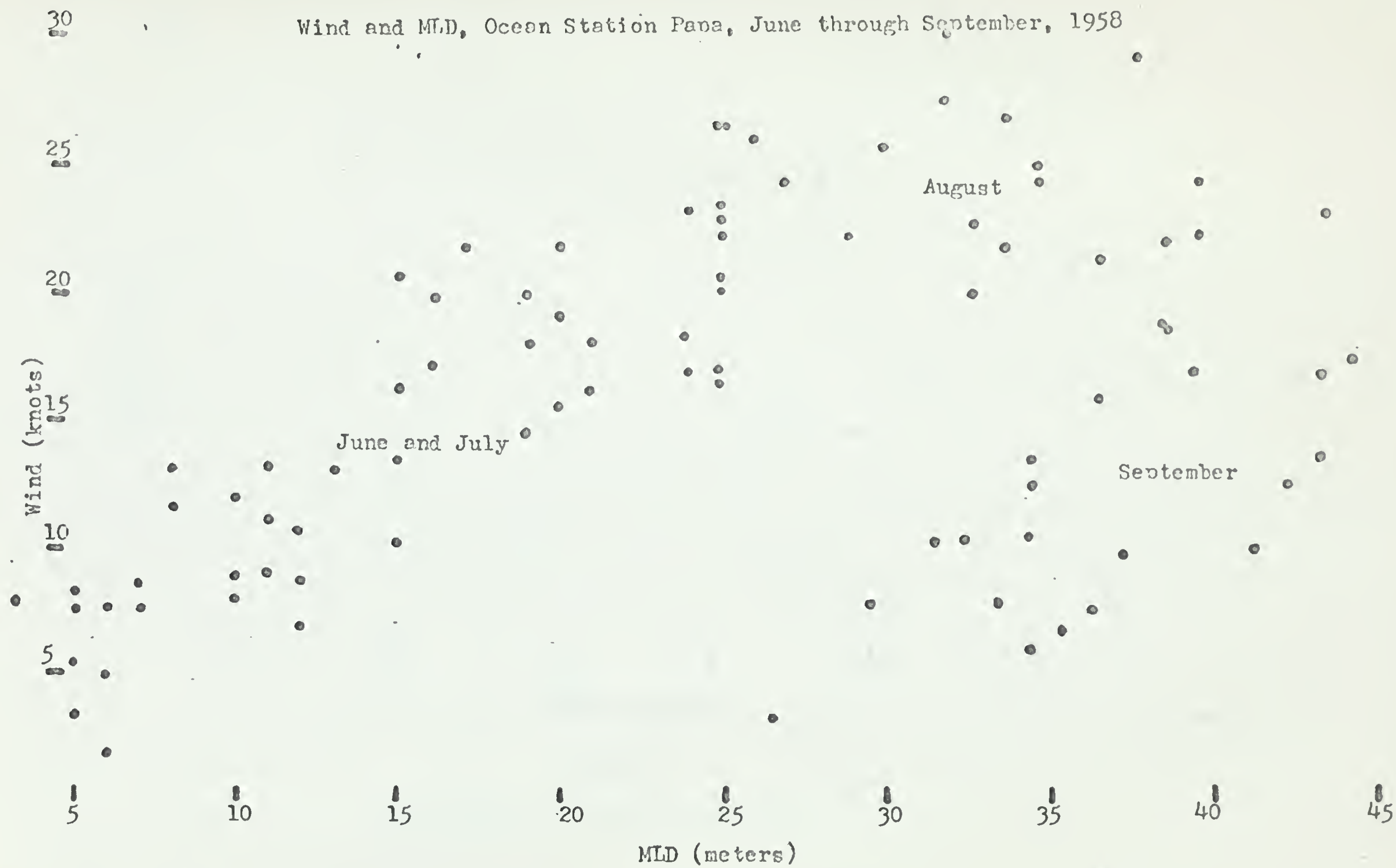
Wind and MLD, Ocean Station Papa, June through September, 1958
Wind (knots)
30
25
20
15
10
5
June and July
August
September
5
10
15
20
25
30
35
40
45
MLD (meters)

Figure 2

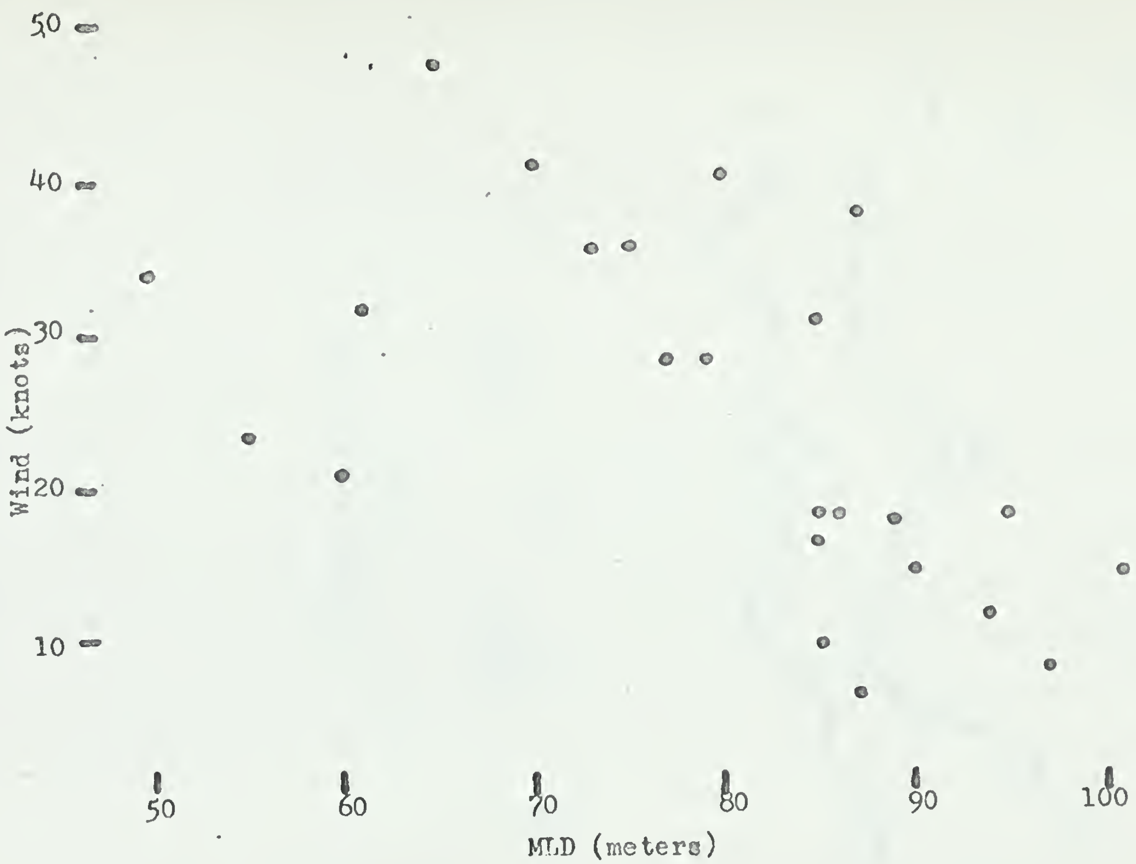

Figure 3

The Effects of Convection on MLD

Figure 4

Lag of MLD from Wind

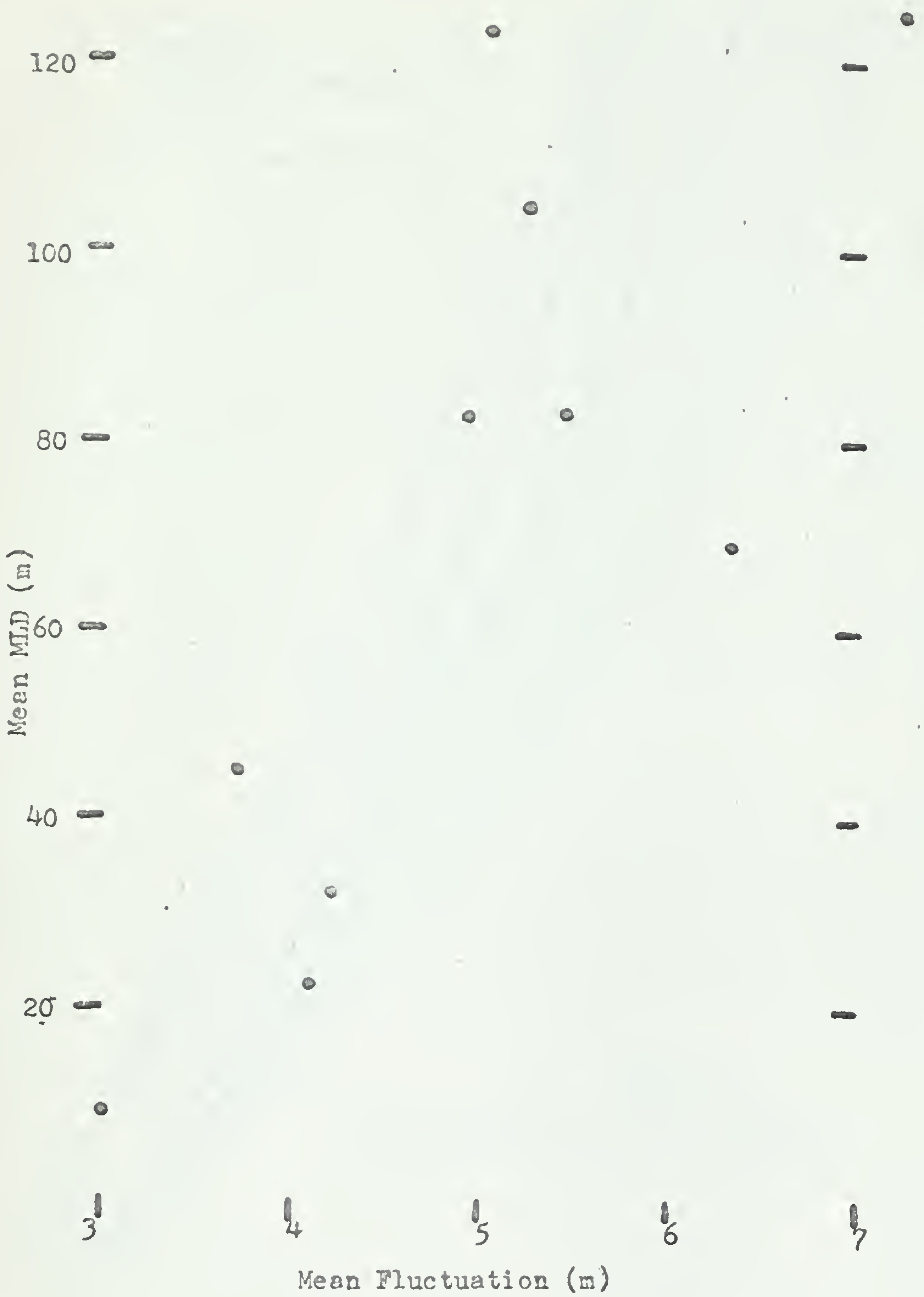

Figure 5

Mean Hourly Fluctuation of MLD

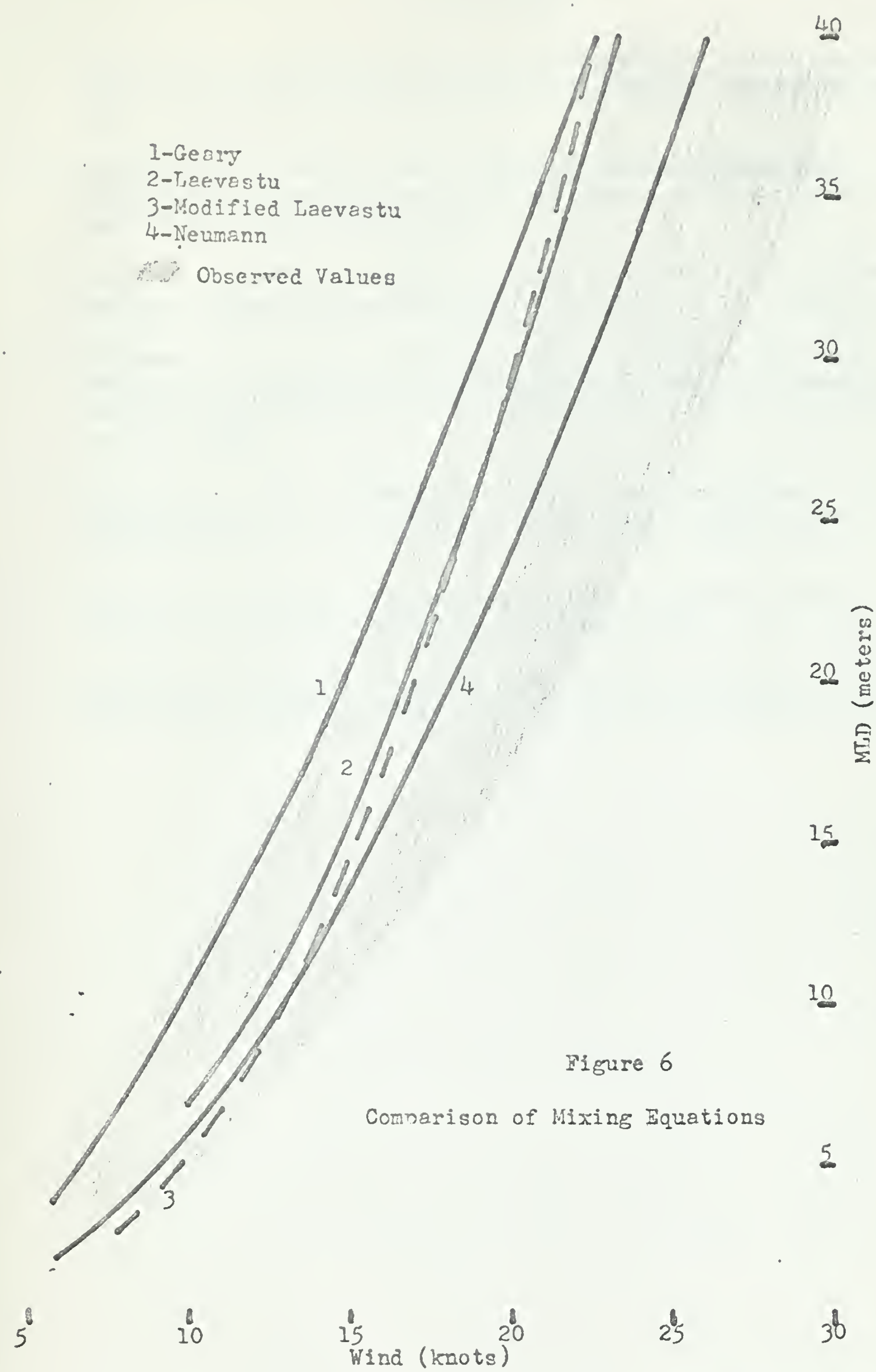

Figure 6

Comparison of Mixing Equations

BIBLIOGRAPHY

1. Laevastu, T., Factors Affecting the Temperature of the Surface Layer of the Sea, Societas Scientiarum Fennica, Commentationes Physico-Mathematicae XXVI, 1960.

2. Fleagle, R. G., Note on the effect of air-sea temperature difference on wave generation, Trans. Amer. Geophys. Union, vol.37, no.3, pp. 275-277, 1956.

3. Munk, W. H. and E. R. Anderson, Notes on a theory of the thermocline, Journal Marine Research, vol.7, no. 3, pp. 276-295, 1948.

4. Pierson, W. J., Jr., G. Neumann, and R. W. James, Practical Methods for Observing and Forecasting Ocean Waves by Means of Wave Spectrum and Statistics, U. S. Hydrographic Office Publication no. 603, 1955.

5. Schule, J. J., Jr., Effects of weather upon the thermal structure of the ocean, U. S. Hydrographic Office Misc. Publication, 15360, 1952.

6. Ball, F. K., Control of inversion height by heating, Quart. J. R. Met. Soc., vol. 86, no.370, pp. 483-494, 1960.

7. Neumann, G., On wind generated wave motion at subsurface levels, Trans. Amer. Geophys, Union, vol. 36, no.6, pp. 985-992, 1955.

APPENDIX I

GEARY'S MODEL

Mechanical mixing of water particles across the thermocline must be inhibited when the vertical forces of particle motion are opposed by the buoyant force of the denser water below the thermocline. Based upon this consideration it is possible to establish a simple model for mechanical mixing by wave action with the following assumptions:

1) A homogeneous layer of water with density ρ (the mixed layer) overlies homogeneous water with a greater density, ρ'. The difference in density between the two layers is caused solely by their temperature difference.
2) Subsurface particle motion is in accordance with simple Airy wave theory for deep water, i.e, the particles move in circular orbits with their diameters decreasing exponentially with increasing depth.
3) Penetration of the thermocline interface stops when the vertical forces of a particle in motion are exactly balanced by the buoyant force. The balance of forces in such a system are given by:

$$\rho g + \rho A \sigma^2 e^{-kz} = \rho' g \tag{2}$$

Where ρg is the gravity force, $\rho A \sigma^2 e^{-kz}$ is the maximum force due to the vertical component of acceleration of the particle in orbital motion, and $\rho' g$ is the buoyant force.

The force balance of (2) is correct, but it must be understood that the acceleration term $A \sigma^2 e^{-kz}$ applies to laminar

conditions of wave motion and is used here only as an approximation to the magnitude of vertical particle acceleration in what must be actually turbulent motion. The laminar acceleration term is utilized in an attempt to resolve in a general way the difficulties of this simple, mechanistic approach.

Equation (2) can be evaluated implicitly for depth as a function of wind speed alone if the stability of the thermocline is assumed constant, i.e., $\frac{\rho'-\rho}{\rho}$ = constant, for all depths and the various surface wave characteristics are calculated according to Neumann's spectrum of wave generation by wind.

The stability term, $\frac{\rho'-\rho}{\rho}$, was evaluated from the data at ocean station Papa for June and July of 1958, and was found to be approximately between the limits of 0.0001 and 0.0004. Using [4] to obtain the wave characteristics $\tilde{\tilde{T}}$, $\tilde{\tilde{L}}$, and $\bar{H}$ for fully developed seas, table 1 gives the computed values of MLD for various windspeeds and two stability values. Expressing (2) as

$$e^{-kz} = \frac{g}{A\sigma^2}\left(\frac{\rho'-\rho}{\rho}\right) , \qquad (3)$$

a sample calculation follows: for a given windspeed $\tilde{\tilde{T}}$, $\tilde{\tilde{L}}$, $\bar{H}$ are specified as above. Assume $\frac{\rho'-\rho}{\rho} = 0.0004$, $A = \frac{\bar{H}}{2}$, $\sigma = \frac{2\pi}{\tilde{\tilde{T}}}$, $k = \frac{2\pi}{\tilde{\tilde{L}}}$; solve (3) for z, which represents the MLD.

Table 1

Geary's MLD Values

Windspeed (knots)	Mixed-Layer Depth (meters)	
	$\frac{\rho'-\rho}{\rho} = 0.0001$	$\frac{\rho'-\rho'}{\rho} = 0.0004$
10	8.8	6.9
12	12.8	10.0
14	17.6	14.0
16	23.3	18.4
18	29.7	23.6
20	36.8	29.4
22	45.0	36.0
24	54.0	43.0
26	63.5	51.0
28	73.7	59.4

APPENDIX II

LAEVASTU'S MODEL

Laevastu [1] gives a tentative formula for the depth of mixing by waves; unfortunately he does not give the derivation, but states that he arrives at the expression by using,

> the relations for trochoid waves for computing the velocities of water particles at various wave height and at various depths, the depths given by Neumann [5], where the total wave energy has decreased to five percent of its value at the sea surface, and assuming that the mixing by waves is negligible at approximately a depth where the diameter of the orbital paths is smaller than 10 centimeters.

The formula is

$$D_m = 12.5\, H_{\frac{1}{3}} \,. \qquad (4)$$

Laevastu uses the following equation for computing significant wave height:

$$H_{\frac{1}{3}} = \frac{0.0008\, V^2 [50 + (T_w - T_a)]}{1 + \left(\frac{5V}{F}\right)\left(1 + \frac{V}{3t_h}\right)} \,. \qquad (5)$$

It is interesting to note that (5) incorporates the sea-air temperature difference parameter, while other formulas for significant wave height do not. Depth of the mixed layer calculated from (4) and (5) is given in table 2, assuming $T_W - T_a = 0$, F = 100 kilometers, and t_h = 24 hours.

MLD obtained from formula (4) is greatly influenced by the equation used to calculate significant wave height. As an illustration of this point, table 3 gives MLD for various wind speeds using formula (4), but with significant wave height computed according to Neumann's equation.

Table 2

Laevastu's MLD Values

Wind speed		Mixed-Layer Depth
meters sec^{-1}	knots	meters
3	5.8	3.9
4	7.8	6.6
5	9.7	9.9
7	13.6	17.6
8	15.5	22.3
9	17.5	26.8
10	19.4	31.8
12	23.3	42.3

Table 3

Modified Laevastu MLD Values

Wind speed		Mixed-Layer Depth
meters sec^{-1}	knots	meters
3	5.8	1.4
4	7.8	2.8
5	9.7	5.0
7	13.6	11.4
8	15.5	16.0
9	17.5	21.5
10	19.4	17.9
12	23.3	44.1

APPENDIX III

NEUMANN'S MODEL

As an extension of his spectrum of wave generation by wind, Neumann [7] derived an equation for the ratio of average potential energy at depth z to the average potential energy of the surface waves as a function of wind speed and depth:

$$\frac{U_z}{U_o} = \left[1 + 4\sqrt{\frac{gz}{V^2}} + \frac{16}{3}\frac{gz}{V^2}\right] e^{-4\sqrt{\frac{gz}{V^2}}}. \tag{6}$$

It is possible to obtain the vertical displacement of a particle at any depth and for a given windspeed by substituting the following relationships into (6):

$$E = \frac{2U}{g\rho}, \tag{7}$$

$$H_{\frac{1}{3}} = 2.83\sqrt{E}. \tag{8}$$

Neumann considers that when the ratio of $\frac{U_z}{U_o}$ becomes equal to 0.05, wave motion is negligible. Solving (6) for the condition where $\frac{U_z}{U_o}$ is equal to 0.05, table 4 is obtained.

Table 4

Neumann's Depth of Negligible Wave Motion

Wind speed meters sec $^{-1}$	Mixed-Layer Depth meters
2	0.42
5	5.7
10	22.9
15	52
20	92
25	143